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## NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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145. Proposed by J. D. WILLIAMS, being the 12th of his fourteen challenge problems proposed in 1832.

Make  $x^2 + y^2 = \square$ ,  $\frac{5}{4}(x^2 + y^2) =$  a cube,  $xy = 2x^3$ ,  $2(x+y) + \frac{xy}{x+y} = \square$ , and  $(x^4 + y^4)(x^2 + y^2) - (x^5 + y^5)\sqrt[4]{(x^2 + y^2)} = \square$ .

No solution has been received.

146. Proposed by PROFESSOR JOSE J. CORONADO, Halapa, Vericruz, Mexico.

Find two numbers whose difference is equal to the difference of their cubes.

I. Solution by G. B. M. ZERR, Philadelphia, Pa.; A. H. HOLMES, Brunswick, Me.; and J. E. SANDERS, Reinerville, O.

Let  $x, y$  be the numbers. Then  $x-y=x^3-y^3$ ,  $1=x^2+xy+y^2$ , if  $x \neq y$ . Let  $x=vy$ .

$$\therefore y = \frac{1}{\sqrt[3]{(v^2+v+1)}}, \quad x = \frac{v}{\sqrt[3]{(v^2+v+1)}}.$$

$$\text{Let } v^2+v+1=(nv+1)^2.$$

$$\therefore v = \frac{1-2n}{n^2-1}. \quad \therefore x = \frac{1-2n}{n^2-n+1}, \text{ and } y = \frac{n^2-1}{n^2-n+1},$$

where  $n$  can have any value, positive or negative, whole or fractional.

II. Solution by DR. L. E. DICKSON, The University of Chicago.

$$x-y=x^3-y^3. \quad \text{Say } x \neq y. \quad \therefore 1=x^2+xy+y^2.$$

If any two numbers are desired, there are an infinitude of answers. If two integers are desired (neither zero), then one must be negative, otherwise  $x^2+xy+y^2 \geq 3$ . Say  $y$  is negative,  $= -z$ .  $\therefore 1=x^2-xz+z^2$ ,  $x$  and  $z$  positive integers;  $\therefore 1-xz=(x-z)^2$ ;  $\therefore 1-xz=0$ , or positive.

$xz \leq 1$ .  $\therefore x=z=1$ . If one is zero, the other=0 or  $\pm 1$ .

$\therefore$  Only integral sets are  $(0, 0)$ ,  $(0, \pm 1)$ ,  $(\pm 1, 0)$ ,  $(\pm 1, \mp 1)$ .

Combined:  $(x, y)$ ,  $\begin{cases} x=0, & \pm 1, \\ y=0, & \pm 1. \end{cases}$

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## MISCELLANEOUS.

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170. Proposed by J. W. NICHOLSON, A. M., LL. D., Baton Rouge, La.

If  $n$  and  $m$  are any two real numbers whatever,  $n$  being less than  $m$ , find a rational  $r$  such that  $\sqrt[n]{n} < r < \sqrt[m]{m}$ .

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

This depends on how we choose our markings. If we choose the natural numbers 1, 2, 3, 4, etc.,  $\sqrt[n]{n}$  may be defined by two infinite series of rational numbers, and  $\sqrt[m]{m}$  may also be so defined. As these two infinite

series cannot be the same when  $n \neq m$ , there is some rational number between them.

Let  $\sqrt{n} = e + f$ , where  $f$  is the decimal part, and  $\sqrt{m} = g + h$ , where  $h$  is the decimal part. Then  $r = e + k$ , where  $k$  is any number from 1 to  $g - e - 1$ .

171. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

If  $\lim_{x \rightarrow a} \frac{\phi(x)}{\psi(x)} = \lambda$ , show  $\lim_{x \rightarrow a} \left[ \frac{\lambda}{\phi(x)} - \frac{1}{\psi(x)} \right] = \frac{\lambda \phi''(a) - \phi'(a)}{2\phi'(a)\psi'(a)}$ .

Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

By hypothesis,  $\phi(a) = 0$ , and  $\psi(a) = 0$ . We must assume that  $\phi'(a) \neq 0$  and  $\psi'(a) \neq 0$ . Then

$$\lim_{x \rightarrow a} \frac{\phi(x)}{\psi(x)} = \left[ \frac{\phi'(x)}{\psi'(x)} \right]_{x=a} = \lambda. \quad \therefore \lambda \psi'(a) - \phi'(a) = 0.$$

$$\begin{aligned} \lim_{x \rightarrow a} \left[ \frac{\lambda}{\phi(x)} - \frac{1}{\psi(x)} \right] &= \lim_{x \rightarrow a} \left[ \frac{\lambda \psi(x) - \phi(x)}{\phi(x)\psi(x)} \right] = \lim_{x \rightarrow a} \left[ \frac{\lambda \psi'(x) - \phi'(x)}{\phi(x)\psi'(x) + \phi'(x)\psi(x)} \right] \\ &= \lim_{x \rightarrow a} \left[ \frac{\lambda \psi''(x) - \phi''(x)}{\phi(x)\psi''(x) + 2\phi'(x)\psi'(x) + \phi''(x)\psi(x)} \right] = \frac{\lambda \psi''(a) - \phi''(a)}{2\phi'(a)\psi'(a)}. \end{aligned}$$

Also solved similarly by G. B. M. Zerr. Unless one assumes that  $\phi(a) = \psi(a) = 0$ , the problem is not true, as may be easily verified. ED. F.



## PROBLEMS FOR SOLUTION.

### ALGEBRA.

297. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

If  $a, b, c, d, f, g, h$  are all real, and  $a, ab - h^2, abc + 2fgh - af^2 - bg^2 - ch^2$  are all positive, show that  $b, c, bc - f^2$ , and  $ca - g^2$  are also positive.

### GEOMETRY.

330. Proposed by J. J. QUINN, Ph. D., New Castle, Pa.

A line pivoted at the origin revolving with a constant angular velocity, intersects another moving parallel to the  $Y$ -axis with a constant linear velocity. (1) Find the locus of their intersection when the ratio of their velocities is as  $m:n$  referred to a quadrant and a radius, respectively. (2) Assume  $m=3$  and  $n=2$ , and apply to the trisection of an angle. (3) Under what conditions will this curve become a quadratrix? (4) Name the curve.